Principles of Chemical Reactivity: Equilibria
Chapter 13

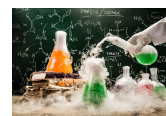Get the CH 223 Companion before lab!

Chemistry 223
Professor Michael Russell
http://mhchem.org/223

Last update: 4/29/24

MAR

CH 223: Lectures and Labs

Lectures: MWF from **9 - 9:50 AM** in **AC 1303** *(this room)*
- Lectures recorded, available soon afterwards
- Lecture notes to print available (under "Problem Sets and Handouts", mhchem.org/223), get **CH 223 Companion** as soon as possible

Labs (Section 01): **Mondays** from **1:10 - 5 PM**
- **Start** in **room AC 2501** ("the recitation")
- Move to **AC 2507** ("the lab") around **3 PM**
- *For first day*, bring a printed copy of the "Determination of an Equilibrium Constant (in class)" Lab (mhchem.org/223), a pair of **safety glasses** (Dollar store ok) and your **calculator**

MAR   *...more on Monday afternoon*

CHEMICAL EQUILIBRIUM

$\rightleftharpoons$ = Bi-directional arrow

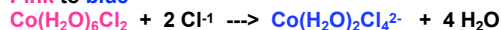$Pb^{2+}(aq) + 2 Cl^-(aq) \longrightarrow PbCl_2(s)$

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Properties of an Equilibrium

Equilibrium systems are
- **DYNAMIC** (in constant motion)
- **REVERSIBLE**
- can be **approached** from **either direction**

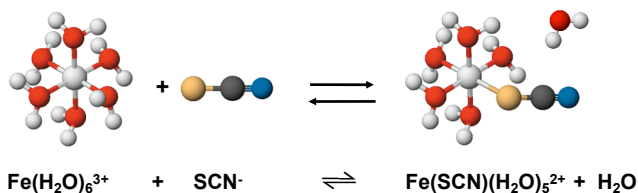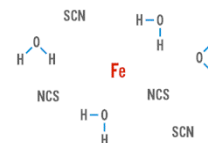**Pink** to **blue**
$Co(H_2O)_6Cl_2 + 2 Cl^{-1} \longrightarrow Co(H_2O)_2Cl_4^{2-} + 4 H_2O$

Blue to **pink**
$Co(H_2O)_2Cl_4^{2-} + 4 H_2O \longrightarrow Co(H_2O)_6Cl_2 + 2 Cl^{-1}$

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Chemical Equilibrium
$Fe^{3+} + SCN^- \rightleftharpoons FeSCN^{2+}$

$Fe(H_2O)_6^{3+} + SCN^- \rightleftharpoons Fe(SCN)(H_2O)_5^{2+} + H_2O$

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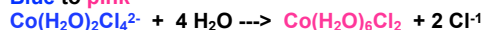
Chemical Equilibrium
$Fe^{3+} + SCN^- \rightleftharpoons FeSCN^{2+}$

After a period of time, the concentrations of reactants and products are constant.
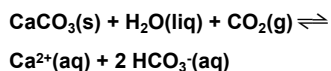The forward and reverse reactions continue after equilibrium is attained.

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Phase changes:
$$H_2O(s) \rightleftharpoons H_2O(liq)$$
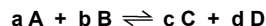
Examples of Chemical Equilibria

Formation of stalactites
and stalagmites:

$$CaCO_3(s) + H_2O(liq) + CO_2(g) \rightleftharpoons$$
$$Ca^{2+}(aq) + 2\ HCO_3^-(aq)$$

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The Equilibrium Constant

For any type of chemical equilibrium of the type

$$a\,A + b\,B \rightleftharpoons c\,C + d\,D$$

the following is a CONSTANT (at a *given T*)

conc. of products

$$K = \frac{[C]^c\,[D]^d}{[A]^a\,[B]^b}$$

equilibrium constant   conc. of reactants

If K is known, we can predict concentrations of products or reactants.

The Equilibrium Constant: Example

Equilibrium constants, **K**, come from *kinetic rate constants, k*

Example: For $2\ NO_{2(g)} \rightleftharpoons N_2O_{4(g)}$,

$$rate_f = k_f[NO_2]^2$$

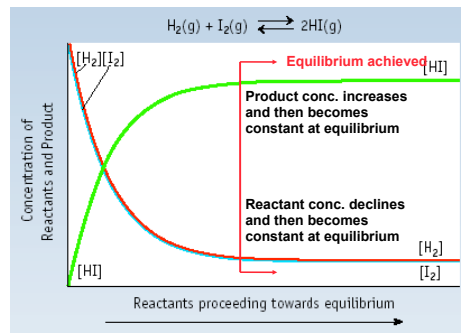$$rate_r = k_r[N_2O_4]$$

When rates equal, **rate$_f$ = rate$_r$**, and

$$k_f[NO_2]^2 = k_r[N_2O_4]$$

$$[N_2O_4]/[NO_2]^2 = k_f/k_r = K \text{ (equilibrium)}$$

Product of two constants (k_f & k_r) is itself a constant (K)!

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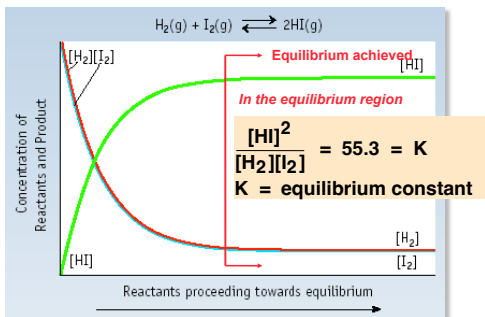
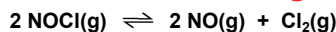## The Equilibrium Constant: Example

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The Equilibrium Constant: Example

In the equilibrium region

$$\frac{[HI]^2}{[H_2][I_2]} = 55.3 = K$$

K = equilibrium constant

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Determining K
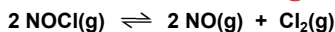
$$2\ NOCl(g) \rightleftharpoons 2\ NO(g) + Cl_2(g)$$

Place 2.00 mol of NOCl is a 1.00 L flask. At equilibrium you find 0.66 mol/L of NO. Calculate K.

Solution

Set of a table of concentrations (ICE)

	[NOCl]	[NO]	[Cl$_2$]
Initial	2.00	0	0
Change			
Equilibrium		0.66	

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Determining K

2 NOCl(g) $\rightleftharpoons$ 2 NO(g) + Cl$_2$(g)

Place 2.00 mol of NOCl is a 1.00 L flask. At equilibrium you find 0.66 mol/L of NO. Calculate K.

Solution

Set of a table of concentrations (ICE)

	[NOCl]	[NO]	[Cl$_2$]
Initial	2.00	0	0
Change	-0.66	+0.66	+0.33
Equilibrium	1.34	0.66	0.33

How to remember ICE:

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Determining K

2 NOCl(g) $\rightleftharpoons$ 2 NO(g) + Cl$_2$(g)
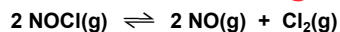
	[NOCl]	[NO]	[Cl$_2$]
Initial	2.00	0	0
Change	-0.66	+0.66	+0.33
Equilibrium	1.34	0.66	0.33

$$K = \frac{[NO]^2[Cl_2]}{[NOCl]^2}$$

$$K = \frac{[NO]^2[Cl_2]}{[NOCl]^2} = \frac{(0.66)^2(0.33)}{(1.34)^2} = 0.080$$

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Writing and Manipulating K Expressions

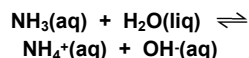**Solids and liquids *NEVER* appear in equilibrium expressions.**

S(s) + O$_2$(g) $\rightleftharpoons$ SO$_2$(g)
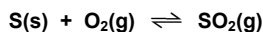
$$K = \frac{[SO_2]}{[O_2]}$$

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Writing and Manipulating K Expressions

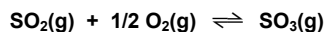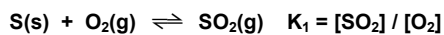**Solids and liquids *NEVER* appear in equilibrium expressions.**

NH$_3$(aq) + H$_2$O(liq) $\rightleftharpoons$ NH$_4^+$(aq) + OH$^-$(aq)
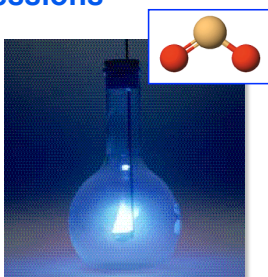
$$K = \frac{[NH_4^+][OH^-]}{[NH_3]}$$

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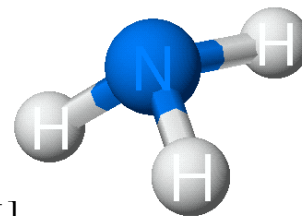
Writing and Manipulating K Expressions

Adding equations for reactions

S(s) + O$_2$(g) $\rightleftharpoons$ SO$_2$(g)   K$_1$ = [SO$_2$] / [O$_2$]

SO$_2$(g) + 1/2 O$_2$(g) $\rightleftharpoons$ SO$_3$(g)

   K$_2$ = [SO$_3$] / [SO$_2$][O$_2$]$^{1/2}$
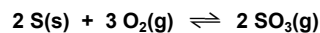
NET EQUATION

S(s) + 3/2 O$_2$(g) $\rightleftharpoons$ SO$_3$(g)
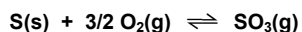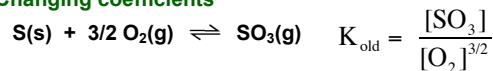
$$K_{net} = \frac{[SO_3]}{[O_2]^{3/2}} = K_1 \cdot K_2$$

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Writing and Manipulating K Expressions

Changing coefficients

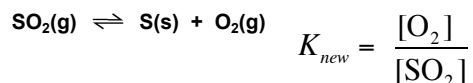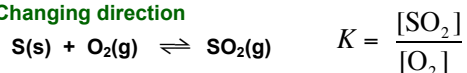S(s) + 3/2 O$_2$(g) $\rightleftharpoons$ SO$_3$(g) $K_{old} = \dfrac{[SO_3]}{[O_2]^{3/2}}$

2 S(s) + 3 O$_2$(g) $\rightleftharpoons$ 2 SO$_3$(g)

$$K_{new} = \frac{[SO_3]^2}{[O_2]^3}$$

$$K_{new} = \frac{[SO_3]^2}{[O_2]^3} = (K_{old})^2$$

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Writing and Manipulating K Expressions

Changing direction

$$S(s) + O_2(g) \rightleftharpoons SO_2(g) \qquad K = \frac{[SO_2]}{[O_2]}$$

$$SO_2(g) \rightleftharpoons S(s) + O_2(g) \qquad K_{new} = \frac{[O_2]}{[SO_2]}$$

$$and: \quad K_{new} = \frac{[O_2]}{[SO_2]} = \frac{1}{K_{old}}$$

See *Manipulating Equilibrium Constant Expressions*

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Kp Expressions

We have been writing K in terms of M (mol/L), designated by K_c

Equilibrium constants expressed in terms of **gases** designated K_p where all pressures in **atm** *(760 mm Hg = 1 atm)*

For: $N_2(g) + 3 H_2(g) \rightleftharpoons 2 NH_3(g)$

$$K_c = \frac{[NH_3]^2}{[N_2][H_2]^3} \qquad K_p = \frac{P_{NH_3}^2}{P_{N_2} P_{H_2}^3}$$

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Converting K_c Into K_p

$$K_p = K_c(RT)^{\Delta n}$$

T = Temperature (K)

R = 0.082057 L atm mol^{-1} K^{-1}

Δn = change in moles of gas

Example:
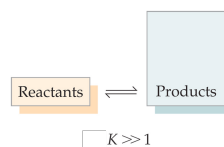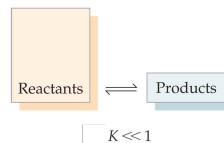$N_2(g) + 3 H_2(g) \rightleftharpoons 2 NH_3(g)$
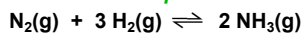
$\Delta n = 2 - 4 = -2$

see: Types of Equilibrium Constants

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The Meaning of K

If *K>>1*, the reaction is *product-favored*; **product** predominates at equilibrium.

If *K<<1*, the reaction is *reactant-favored*; **reactant** predominates at equilibrium.

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The Meaning of K

Can tell if a reaction is product-favored or reactant-favored.

For: $N_2(g) + 3 H_2(g) \rightleftharpoons 2 NH_3(g)$

$$K_c = \frac{[NH_3]^2}{[N_2][H_2]^3} = 3.5 \times 10^8$$

Conc. of products is **much greater** than that of reactants at equilibrium.

The reaction is strongly **product-favored**.

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The Meaning of K

For: $AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq)$

$K_c = [Ag^+][Cl^-] = 1.8 \times 10^{-5}$

Conc. of products is **much less** than that of reactants at equilibrium.
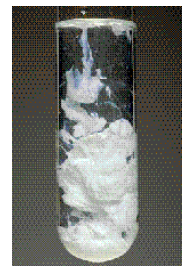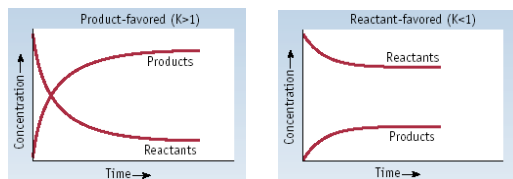
The reaction is strongly **reactant-favored**.

The reverse reaction
$Ag^+(aq) + Cl^-(aq) \rightleftharpoons AgCl(s)$
is product-favored,

$K_{rev} = 1/1.8 \times 10^{-5} = 5.6 \times 10^4$

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Product- or Reactant Favored

Product-favored (K>1)	Reactant-favored (K<1)

Product-favored
K > 1

Reactant-favored
K < 1

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The Reaction Quotient, Q

All reacting chemical systems can be characterized by their REACTION QUOTIENT, Q.

For: $aA + bB \rightleftharpoons cC + dD$

Under Any Reaction Conditions

$$\text{Reaction quotient} = Q = \frac{[C]^c[D]^d}{[A]^a[B]^b} \quad \begin{array}{l}\text{Product concentrations} \\ \\ \text{Reactant concentrations}\end{array}$$

If Q = K, then system is at equilibrium.
If Q ≠ K, then system is not at equilibrium

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The Reaction Quotient, Q

At *any point* in the reaction
$H_2 + I_2 \rightleftharpoons 2 HI$

$$Q = \text{reaction quotient} = \frac{[HI]^2}{[H_2][I_2]}$$
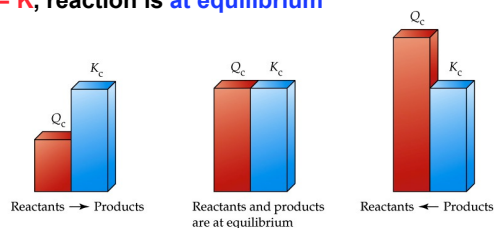
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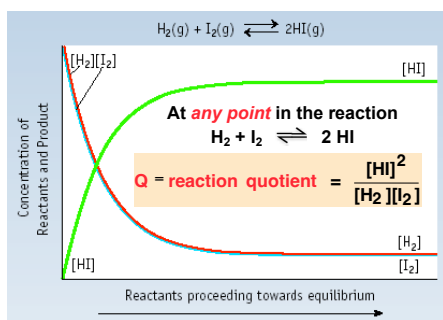
The Reaction Quotient, Q

Comparing Q and K:
Q < K, reaction shifts to the products (right)
Q > K, reaction shifts to the reactants (left)
Q = K, reaction is at equilibrium

Reactants → Products   Reactants and products are at equilibrium   Reactants ← Products
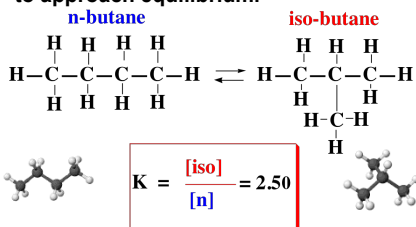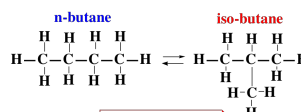
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The Reaction Quotient, Q

We can use Q to tell if a reaction is at equilibrium. If not at equilibrium, we can predict which way the reaction will move to approach equilibrium.

n-butane   **iso-butane**

$$K = \frac{[iso]}{[n]} = 2.50$$

see Understanding K and Q Handout

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The Reaction Quotient, Q

n-butane   **iso-butane**

$$K = \frac{[iso]}{[n]} = 2.50$$

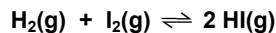If [iso] = 0.35 M and [n] = 0.15 M, are you at equilibrium?
Which way does the reaction "shift" to approach equilibrium?

$$Q = \frac{\text{conc. of iso}}{\text{conc. of n}} = \frac{0.35}{0.15} = 2.3$$
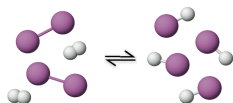
To [iso]!   Q = 2.3, Q < K, reaction will shift to "right" (product side, or [iso])

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Typical Calculations

$H_2(g) + I_2(g) \rightleftharpoons 2 HI(g)$

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = 55.3$$

Place 1.00 mol each of H_2 and I_2 in a 1.00 L flask. Calc. equilibrium concentrations.

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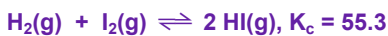**$H_2(g) + I_2(g) \rightleftharpoons 2 HI(g)$, $K_c = 55.3$**

Step 1. Set up ICE table to define EQUILIBRIUM concentrations.

	$[H_2]$	$[I_2]$	$[HI]$
Initial			
Change			
Equilib			

where x is defined as am't of H_2 and I_2 consumed on approaching equilibrium.

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$H_2(g) + I_2(g) \rightleftharpoons 2 HI(g)$, $K_c = 55.3$

Step 2. Put equilibrium concentrations into K_c expression.

$$K_c = \frac{[2x]^2}{[1.00-x][1.00-x]} = 55.3$$

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$H_2(g) + I_2(g) \rightleftharpoons 2 HI(g)$, $K_c = 55.3$

Step 3. Solve K_c expression - take square root of both sides.

$$7.44 = \frac{2x}{1.00 - x}$$
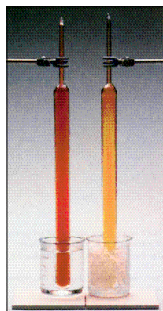
7.44 (1.00 - x) = 2x

7.44 - 7.44x = 2x

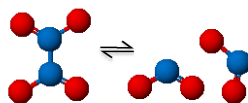**x = 7.44 / 9.44 = 0.788**

Therefore, at equilibrium

$[H_2] = [I_2] = 1.00 - x = 0.21$ M

$[HI] = 2x = 1.58$ M

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Nitrogen Dioxide Equilibrium

$N_2O_4(g) \rightleftharpoons 2 NO_2(g)$

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Nitrogen Dioxide Equilibrium
$N_2O_4(g) \rightleftharpoons 2 NO_2(g)$

$$K_c = \frac{[NO_2]^2}{[N_2O_4]} = 0.0059 \text{ at } 298 \text{ K}$$

If initial concentration of N_2O_4 is 0.50 M, what are the equilibrium concentrations?

Step 1. Set up an equilibrium table

	$[N_2O_4]$	$[NO_2]$
Initial	0.50	0
Change		
Equilib		

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Nitrogen Dioxide Equilibrium
$N_2O_4(g) \rightleftharpoons 2\ NO_2(g)$

$$K_c = \frac{[NO_2]^2}{[N_2O_4]} = 0.0059 \text{ at } 298 \text{ K}$$

If initial concentration of N_2O_4 is 0.50 M, what are the equilibrium concentrations?

Step 1. Set up an equilibrium table

	$[N_2O_4]$	$[NO_2]$
Initial	0.50	0
Change	-x	+2x
Equilib	0.50 - x	2x

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Nitrogen Dioxide Equilibrium
$N_2O_4(g) \rightleftharpoons 2\ NO_2(g)$

Step 2. Substitute into K_c expression and solve.

$$K_c = 0.0059 = \frac{[NO_2]^2}{[N_2O_4]} = \frac{(2x)^2}{(0.50 - x)}$$

Rearrange: 0.0059 (0.50 - x) = 4x²

0.0029 - 0.0059x = 4x²

4x² + 0.0059x - 0.0029 = 0

This is a QUADRATIC EQUATION

ax² + bx + c = 0

a = 4   b = 0.0059   c = -0.0029

*CH 223: report **both** values of x when performing quadratic calculations*

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Nitrogen Dioxide Equilibrium
$N_2O_4(g) \rightleftharpoons 2\ NO_2(g)$

Solve the quadratic equation for x.

ax² + bx + c = 0

a = 4   b = 0.0059   c = -0.0029

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0.0059 \pm \sqrt{(0.0059)^2 - 4(4)(-0.0029)}}{2(4)}$$

x = -0.00074 ± 1/8(0.046)^{1/2} = -0.00074 ± 0.027

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Nitrogen Dioxide Equilibrium
$N_2O_4(g) \rightleftharpoons 2\ NO_2(g)$

$$x = \frac{-0.0059 \pm \sqrt{(0.0059)^2 - 4(4)(-0.0029)}}{2(4)}$$

x = -0.00074 ± 1/8(0.046)^{1/2} = -0.00074 ± 0.027

x = 0.026 or -0.028

But a *negative* value is not reasonable.

***Conclusion:* x = 0.026**

$[N_2O_4]$ = 0.50 - x = 0.47 M

$[NO_2]$ = 2x = 0.052 M

*CH 223: report **both** values of x when performing quadratic calculations*

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Le Chatelier's Principle

Temperature, changes in volume, changes in pressure and changes in concentration affect equilibria.

The outcome is governed by LE CHATELIER'S PRINCIPLE

"...if a system at equilibrium is disturbed, the system tends to shift its equilibrium position to counter the effect of the disturbance."
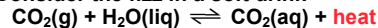
Henri Le Chatelier, 1850 - 1936

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Temperature effects change the numeric value of K

Consider the fizz in a soft drink
$CO_2(g) + H_2O(liq) \rightleftharpoons CO_2(aq) + $ **heat**

Decrease T. *What happens to equilibrium position? To value of K?*

K = $[CO_2]$ / P (CO_2)

K *increases* as T goes down because $[CO_2]$ increases *and* P(CO_2) decreases.

Increase T. *Now what?*

Equilibrium shifts left *and* **K** *decreases.*

see: Le Chatelier's Guide

MAR

Temperature Effects on Equilibrium

N_2O_4 (colorless) + heat $\rightleftharpoons$ 2 NO_2 (brown)

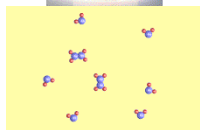ΔH^o = + 57.2 kJ *(endothermic)*
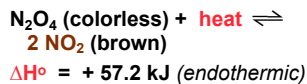
$$K_c = \frac{[NO_2]^2}{[N_2O_4]}$$

K_c = 0.00077 at 273 K
K_c = 0.0059 at 298 K

K changes with temperature

MAR

Volume & Pressure Effects on Equilibrium

N_2O_4(g) + heat $\rightleftharpoons$ 2 NO_2(g)
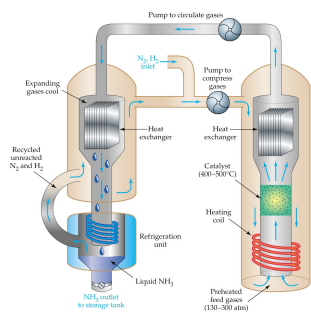*colorless gas   brown gas*
endothermic
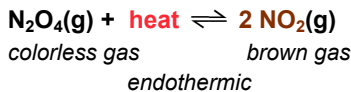
Volume and **Pressure** changes affect equilibrium

Larger volume / Less Pressure = more molecules
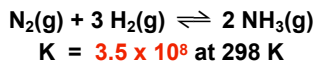
Smaller volume / Higher Pressure = fewer molecules

MAR

NH_3 Production

Catalyst for the Haber-Frisch reaction is mostly Fe with some KOH

N_2(g) + 3 H_2(g) $\rightleftharpoons$ 2 NH_3(g)
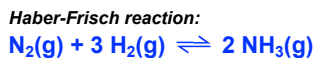**K = 3.5 x 10^8 at 298 K**

MAR

EQUILIBRIUM AND EXTERNAL EFFECTS

Concentration changes ---> **no change in K** - only the *position* of equilibrium changes.

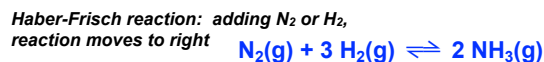We can use the "water U-tube" to demonstrate this phenomena (and connect to Haber-Frisch)
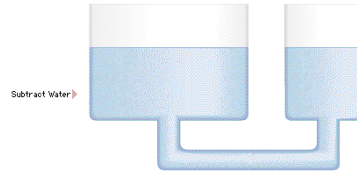
Haber-Frisch reaction:
N_2(g) + 3 H_2(g) $\rightleftharpoons$ 2 NH_3(g)

Water U-tube

MAR

Le Chatelier's Principle

Adding a "reactant" to a chemical system.

Haber-Frisch reaction: adding N_2 or H_2, reaction moves to right
N_2(g) + 3 H_2(g) $\rightleftharpoons$ 2 NH_3(g)

MAR

Le Chatelier's Principle

Removing a "reactant" from a chemical system.

Haber-Frisch reaction: removing N_2 or H_2, reaction moves to left
N_2(g) + 3 H_2(g) $\rightleftharpoons$ 2 NH_3(g)

MAR

Le Chatelier's Principle

Add Water
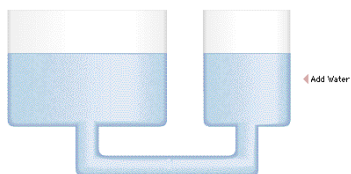
Adding a "product" to a chemical system.

Haber-Frisch reaction: adding NH₃,
reaction moves to left

$$N_2(g) + 3\ H_2(g) \rightleftharpoons 2\ NH_3(g)$$

MAR

Le Chatelier's Principle

Subtract Water

Removing a "product" from a chemical system.

Haber-Frisch reaction: removing NH₃,
reaction moves to right

$$N_2(g) + 3\ H_2(g) \rightleftharpoons 2\ NH_3(g)$$

MAR

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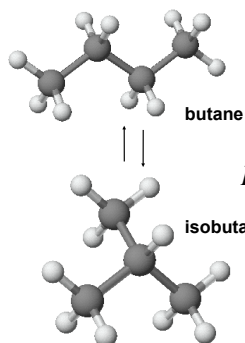
butane

isobutane

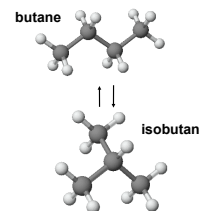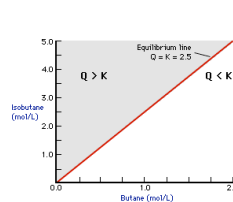## Butane-Isobutane Equilibrium

$$K = \frac{[\text{isobutane}]}{[\text{butane}]} = 2.50$$

MAR

Butane ⇌ Isobutane

Assume you are at equilibrium with [iso] = 1.25 M and [butane] = 0.50 M. Now add 1.50 M butane. When the system comes to equilibrium again, what are [iso] and [butane]? K = 2.50

butane

isobutane

MAR

Butane ⇌ Isobutane

Assume you are at equilibrium with [iso] = 1.25 M and [butane] = 0.50 M. Now add 1.50 M butane. When the system comes to equilibrium again, what are [iso] and [butane]? K = 2.50

Solution

Calculate Q immediately after adding more butane and compare with K.

$$Q = \frac{[\text{isobutane}]}{[\text{butane}]} = \frac{1.25}{0.50 + 1.50} = 0.625$$

Q is *LESS THAN* K. Therefore, the reaction will shift to the _____.

MAR

Butane ⇌ Isobutane

You are at equilibrium with [iso] = 1.25 M and [butane] = 0.50 M. Now add 1.50 M butane.

Solution

Q is less than K, so equilibrium shifts right - away from butane and toward isobutane.

Set up concentration (ICE) table

	[butane]	[isobutane]
Initial		
Change		
Equilibrium		

MAR

Butane ⇌ Isobutane

You are at equilibrium with [iso] = 1.25 M and [butane] = 0.50 M. Now add 1.50 M butane.

Solution

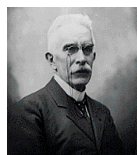$$K = 2.50 = \frac{[\text{isobutane}]}{[\text{butane}]} = \frac{1.25 + x}{2.00 - x}$$

x = 1.07 M

At the new equilibrium position,

[butane] = 0.93 M and [isobutane] = 2.32 M

Equilibrium has shifted toward isobutane.

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Le Chatelier's Principle - Overview

Change T
- changes the value of K
- changes observed in P, V or concentrations at equilibrium

Add or take away reactant or product
- K does not change
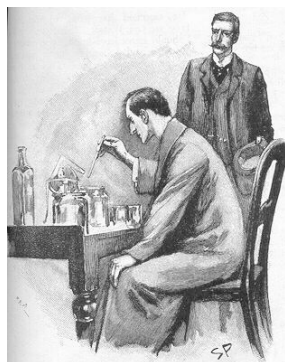- Reaction adjusts to new equilibrium "position"

Use a catalyst
- reaction comes more quickly to equilibrium. K not changed.

MAR

End of Chapter 13

See:
- *Chapter Thirteen Study Guide*
- *Chapter Thirteen Concept Guide*
- *Types of Equilibrium Constants*
- *Important Equations (following this slide)*
- *End of Chapter Problems (following this slide)*

MAR

Important Equations, Constants, and Handouts from this Chapter:

for: $a A + b B \rightleftharpoons c C + d D$

$$K = \frac{[C]^c\,[D]^d}{[A]^a\,[B]^b}$$

conc. of products

conc. of reactants

equilibrium constant

$K_p = K_c(RT)^{\Delta n}$

R = 0.082057 L atm mol^{-1} K^{-1}

Under Any Reaction Conditions

$$\text{Reaction quotient} = Q = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

Product concentrations

Reactant concentrations

Handouts:
- *Manipulating Equilibrium Constant Expressions*
- *Types of Equilibrium Constants*

Le Chatelier's Principle

MAR

End of Chapter Problems: *Test Yourself*

1. Write an equilibrium constant expression for the following reaction: $C(s) + CO_2(g) \rightleftharpoons 2\,CO(g)$

2. K = 5.6 x 10^{-12} at 500 K for the dissociation of iodine molecules to iodine atoms: $I_2(g) \rightleftharpoons 2\,I(g)$ A mixture has [I_2] = 0.020 mol/L and [I] = 2.0 x 10^{-8} mol/L. Is the reaction at equilibrium (at 500 K)? If not, which way must the reaction proceed to reach equilibrium?

3. The reaction: $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ was examined at 250 °C. At equilibrium, [PCl_5] = 4.2 x 10^{-5} mol/L, [PCl_3] = 1.3 x 10^{-2} mol/L, and [Cl_2] = 3.9 x 10^{-3} mol/L. Calculate K for the reaction.

4. The equilibrium constant for the dissociation of iodine molecules to iodine atoms: $I_2(g) \rightleftharpoons 2\,I(g)$ is 3.76 x 10^{-3} at 1000 K. Suppose 0.105 mol of I_2 is placed in a 12.3 L flask at 1000 K. What are the concentrations of I_2 and I when the system comes to equilibrium?

5. Dinitrogen trioxide decomposes to NO and NO_2 in an endothermic process (ΔH = 40.5 kJ/mol): $N_2O_3(g) \rightleftharpoons NO(g) + NO_2(g)$ Predict the effect of the following changes on the position of the equilibrium (left, right, or no change):
   a. adding more $N_2O_3(g)$
   b. adding more $NO_2(g)$
   c. increasing the volume of the reaction flask
   d. lowering the temperature

MAR

End of Chapter Problems: *Answers*

1. K_c = [CO]^2 / [CO$_2$] or K_p = P_{CO}^2 / P_{CO2}
2. No, Q < K, reaction will proceed to the right (more products)
3. K = 1.2
4. [I_2] = 0.00614 M, [I] = 0.00480 M
5. a. right b. left c. right d. left

MAR