CH 222 Chapter Twenty-one Concept Guide

1. Terminology

Alpha Radiation (α):	Alpha particles are positively charged particles ejected at high speed from certain radioactive substances; a helium nucleus
Beta Radiation (β):	Beta particles are electrons that are ejected at high speed from certain radioactive substances
Gamma Radiation (γ):	High-energy electromagnetic radiation
Nuclear Reaction:	A reaction involving one or more atomic nuclei, resulting in a change in the identities of the isotopes
Nucleons:	A nuclear particle, either a neutron or a proton
Radioactive Decay Series:	A series of nuclear reactions by which a radioactive isotopes decays to form a stable isotope
Positrons:	A nuclear particle having the same mass as an electron but a positive charge
Nuclear Binding Energy:	The energy required to separate a nucleus into its individual nucleons
Fission:	The highly exothermic process by which very heavy nuclei split to form lighter nuclei
Fusion:	the state change from solid to liquid
Half-life:	The time required for the concentration of one of the reactants to reach half of its initial value
Activity (A):	A measure of the rate of nuclear decay, the number of disintegrations observed in a sample per unit cell
Nuclear Reactor:	A container in which a controlled nuclear reaction occurs
Nuclear Fusion:	The highly exothermic process by which comparatively light nuclei combine to form heavier nuclei
Plasma:	A gas like phase of matter that consists of charged particles

Röntgen	A unit of radiation dosage proportional to the amount of ionization produced in air
Rad:	A unit of radiation dosage which measures the radiation dose to living tissue
Rem:	A unit of radiation dosage which takes into account the differing intensities of different radiation (alpha, beta and gamma) upon human tissue
Curie (Ci):	A unit of radioactivity which measures activity. One curie represents any radioactive isotope which undergoes 3.7 x 10^{10} disintegrations per second (dps).

2. α Radioactive Decay Series

Problem

1. A radioactive decay series begins with ${}^{235}_{92}U$ and ends with ${}^{207}_{82}Pb$. What is the total number of α and β particles emitted in this series?
2. The first three steps of this series involve (in order) α, β, and α emissions. Write the nuclear equations for each of these steps.

Solution

1. Mass declines by 28 mass units (235-207) in this series. Because a decrease in mass can only occur with α emission, we conclude that seven α particles must be emitted. For each α emission, the atomic number must decrease by 2, so emission of seven α particles causes the atomic number to decrease by 14. The actual decrease in atomic number is 10, however (92-82). Four β particles cause the atomic number to increase by 4. This radioactive decay sequence involves loss of seven α and four β particles.

2. Step 1. ${}^{235}_{92}U \rightarrow {}^{231}_{90}Th + {}^{4}_{2}\alpha$

Step 2. ${}^{231}_{90}Th \rightarrow {}^{231}_{91}Pa + {}^{0}_{-1}\beta$

Step 3 ${}^{231}_{91}Pa \rightarrow {}^{227}_{89}Ac + {}^{4}_{2}\alpha$

3. Balancing Nuclear Reaction

Problem

Complete the following equations. Give the symbol, mass number, and atomic number of the species indicated by the question mark.

1. ${}^{37}_{18}Ar$ + ${}^{0}_{-1}e \rightarrow$?
2. ${}^{11}_{6}C \rightarrow {}^{11}_{5}B$ + ?
3. ${}^{35}_{16}S \rightarrow {}^{35}_{17}Cl$ + ?
4. ${}^{30}_{15}P \rightarrow +{}^{0}_{1}\beta$ + ?

Solution

1. This is an electron capture reaction. The product has a mass number of 37+0 = 37 and an atomic number of 18-1 = 17. The symbol for the product is $\mathbf{{}^{37}_{17}Cl}$.
2. This reaction is recognized as positron ($\mathbf{{}^{0}_{+1}\beta}$) emission. By choosing this particle, the sum of the atomic numbers (6 = 5+1) and the mass numbers (11) on either side of the reaction are equal.
3. Beta ($\mathbf{{}^{0}_{-1}\beta}$) emission is required to balance the mass numbers (35) and atomic numbers (16 = 17-1) on both sides of the equation.
4. The product nucleus is $\mathbf{{}^{30}_{14}Si}$. This balances the mass numbers (30) and atomic numbers (15 = 1+14) on both sides of the equation.

4. Binding Energy

Approach

Einstein's equation from the theory of special relativity states that the energy of a body is equivalent to the mass times the speed of light squared

$$\Delta E = (\Delta m)c^2$$

When comparing nuclear stabilities, scientists generally calculate the binding energy (E_b) per nucleon:

$$\frac{E_b}{\text{Mol nucleons}}$$

where the number of nucleons equals the number of protons plus the number of neutrons in an atom. The binding energy is related to the change in energy by

$$\Delta E = -E_b$$

Problem

Calculate the binding energy (in kJ/mole) and the binding energy per nucleon (in kJ/mole nucleons) for carbon-12.

Solution

The following reaction results in formation of carbon-12:

$$6\ {}^{1}_{1}H + 6\ {}^{1}_{0}n \rightarrow {}^{12}_{6}C$$

The mass of ${}^{1}_{1}H$ is 1.00783 g/mol and the mass of ${}^{1}_{0}n$ is 1.00867 g/mole. Carbon-12, ${}^{12}_{6}C$, is the standard for the atomic masses in the periodic table, and its mass is defined as exactly 12.000 g/mol.

To determine binding energy we must first determine the difference in mass of the products and reactants in this reaction:

$$\Delta m = 12.0000000 - [(6 \times 1.00783) + (6 \times 1.00867)]$$
$$= -9.9000 \times 10^{-2} \text{ g/mol}$$

The binding energy is calculated using $\Delta E = (\Delta m)c^2$.

Using the **mass** in **kilograms** and the **speed of light** in **meters per second** gives an answer for the binding energy in joules:

$$E_b = -(\Delta m)c^2 = -(9.9 \times 10^{-5} \text{ kg/mol})(3.00 \times 10^{8} \text{ m/s})^2$$
$$= 8.91 \times 10^{12} \text{ J/mol} = 8.91 \times 10^{9} \text{ kJ/mol}$$

The **binding energy per nucleon** is determined by dividing the binding energy by the number of nucleons, which in this instance is 12.

$$\frac{E_b}{\text{mol nucleons}} = \frac{8.91 \times 10^{9} \text{ kJ mol}^{-1}}{12 \text{ nucleons mol}^{-1}}$$

$$= \mathbf{7.43 \times 10^{8}\ kJ/mol\ nucleons}$$

5. Rate of Radioactive Decay and Half-Life

Approach

Radioactive decay processes always follow first order kinetics. The activity (A) of a nuclear decay process is proportional to the number of radioactive atoms present (N), or

$$A/A_0 = k(N/N_0)$$

where k equals the decay or rate constant. The first order integrated rate law equation relates the period over which a sample is observed (t) to the fraction of radioactive atoms present after that amount of time has passed

$$\ln N/N_0 = -kt$$

Another convenient method to find the decay constant is through the **half-life**, $t_{1/2}$. Half-life is defined as the time required for the concentration of a reactant to reach half of its initial value (i.e. $N/N_0 = 0.5$). An equation to determine the half-life from the rate constant can be derived from the first order integrated rate law; the results can be expressed as follow:

$$t_{1/2} = 0.693 / k$$

Example

Some high-level radioactive waste with a half-life $t_{1/2}$ of 200.0 years is stored in underground tanks. What time is required to reduce an activity of 6.50×10^{12} disintegrations per minute (dpm) to a fairly harmless activity of 3.00×10^{-3} dpm?

Solution

The data provides the initial activity ($A_o = 6.50 \times 10^{12}$ dpm) and the activity after some elapsed time ($A = 3.00 \times 10^{-3}$ dpm). To find the elapsed time t, first find k from the half-life:

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{200.\ \text{years}} = 0.00347\ \text{year}^{-1}$$

With k known, the time t can be calculated:

$$\ln\left[\frac{3.00 \times 10^{-3}}{6.50 \times 10^{12}}\right] = -[0.00347\ \text{year}^{-1}]t$$

$$-35.312 = [0.00347\ \text{year}^{-1}]t$$

$$t = \frac{-35.312}{-[0.00347\ \text{year}^{-1}]}$$

$$= 1.02 \times 10^{4}\ \text{years}$$

Note that the time unit t and rate constant k must share common units (i.e. years and years^{-1}) for this equation to work properly.