

CH 222 Chapter Twenty-one Concept Guide

1. Terminology

Alpha Radiation (α):	Alpha particles are positively charged particles ejected at high speed from certain radioactive substances; a helium nucleus
Beta Radiation (β):	Beta particles are electrons that are ejected at high speed from certain radioactive substances
Gamma Radiation (γ):	High-energy electromagnetic radiation
Nuclear Reaction:	A reaction involving one or more atomic nuclei, resulting in a change in the identities of the isotopes
Nucleons:	A nuclear particle, either a neutron or a proton
Radioactive Decay Series:	A series of nuclear reactions by which a radioactive isotope decays to form a stable isotope
Positrons:	A nuclear particle having the same mass as an electron but a positive charge
Nuclear Binding Energy:	The energy required to separate a nucleus into its individual nucleons
Fission:	The highly exothermic process by which very heavy nuclei split to form lighter nuclei
Fusion:	the state change from solid to liquid
Half-life:	The time required for the concentration of one of the reactants to reach half of its initial value
Activity (A):	A measure of the rate of nuclear decay, the number of disintegrations observed in a sample per unit cell
Nuclear Reactor:	A container in which a controlled nuclear reaction occurs
Nuclear Fusion:	The highly exothermic process by which comparatively light nuclei combine to form heavier nuclei
Plasma:	A gas like phase of matter that consists of charged particles

Röntgen	A unit of radiation dosage proportional to the amount of ionization produced in air
Rad:	A unit of radiation dosage which measures the radiation dose to living tissue
Rem:	A unit of radiation dosage which takes into account the differing intensities of different radiation (alpha, beta and gamma) upon human tissue
Curie (Ci):	A unit of radioactivity which measures activity. One curie represents any radioactive isotope which undergoes 3.7×10^{10} disintegrations per second (dps).

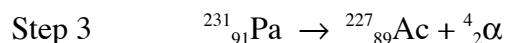
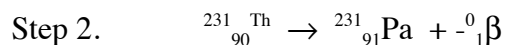
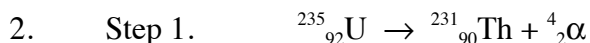
2. α Radioactive Decay Series

Problem

1. A radioactive decay series begins with ${}^{235}_{92}\text{U}$ and ends with ${}^{207}_{82}\text{Pb}$. What is the total number of α and β particles emitted in this series?
2. The first three steps of this series involve (in order) α , β , and α emissions. Write the nuclear equations for each of these steps.

Solution

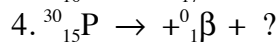
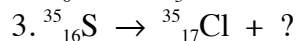
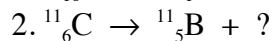
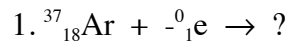
1. Mass declines by 28 mass units (235-207) in this series. Because a decrease in mass can only occur with α emission, we conclude that seven α particles must be emitted. For each α emission, the atomic number must decrease by 2, so emission of seven α particles causes the atomic number to decrease by 14. The actual decrease in atomic number is 10, however (92-82). Four β particles cause the atomic number to increase by 4. This radioactive decay sequence involves loss of seven α and four β particles.



3. Balancing Nuclear Reaction

Problem

Complete the following equations. Give the symbol, mass number, and atomic number of the species indicated by the question mark.



Solution

1. This is an electron capture reaction. The product has a mass number of $37+0 = 37$ and an atomic number of $18-1 = 17$. The symbol for the product is ${}^{37}_{17}\text{Cl}$.
2. This reaction is recognized as positron (${}^{0}_{+1}\beta$) emission. By choosing this particle, the sum of the atomic numbers ($6 = 5+1$) and the mass numbers (11) on either side of the reaction are equal.
3. Beta (${}^{0}_{-1}\beta$) emission is required to balance the mass numbers (35) and atomic numbers ($16 = 17-1$) on both sides of the equation.
4. The product nucleus is ${}^{30}_{14}\text{Si}$. This balances the mass numbers (30) and atomic numbers ($15 = 1+14$) on both sides of the equation.

4. Binding Energy

Approach

Einstein's equation from the theory of special relativity states that the energy of a body is equivalent to the mass times the speed of light squared

$$\Delta E = (\Delta m)c^2$$

When comparing nuclear stabilities, scientists generally calculate the binding energy (E_b) per nucleon:

$$\frac{E_b}{\text{Mol nucleons}}$$

where the number of nucleons equals the number of protons plus the number of neutrons in an atom. The binding energy is related to the change in energy by

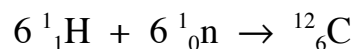
$$\Delta E = -E_b$$

Problem

Calculate the binding energy (in kJ/mole) and the binding energy per nucleon (in kJ/mole nucleons) for carbon-12.

Solution

The following reaction results in formation of carbon-12:



The mass of ${}^1_1\text{H}$ is 1.00783 g/mol and the mass of ${}^1_0\text{n}$ is 1.00867 g/mole. Carbon-12, ${}^{12}_6\text{C}$, is the standard for the atomic masses in the periodic table, and its mass is defined as exactly 12.000 g/mol.

To determine binding energy we must first determine the difference in mass of the products and reactants in this reaction:

$$\begin{aligned}\Delta m &= 12.0000000 - [(6 \times 1.00783) + (6 \times 1.00867)] \\ &= -9.9000 \times 10^{-2} \text{ g/mol}\end{aligned}$$

The binding energy is calculated using $\Delta E = (\Delta m)c^2$.

Using the **mass in kilograms** and the **speed of light in meters per second** gives an answer for the binding energy in joules:

$$\begin{aligned}E_b &= -(\Delta m)c^2 = -(9.9 \times 10^{-5} \text{ kg/mol})(3.00 \times 10^8 \text{ m/s})^2 \\ &= 8.91 \times 10^{12} \text{ J/mol} = 8.91 \times 10^9 \text{ kJ/mol}\end{aligned}$$

The **binding energy per nucleon** is determined by dividing the binding energy by the number of nucleons, which in this instance is 12.

$$\begin{aligned}\frac{E_b}{\text{mol nucleons}} &= \frac{8.91 \times 10^9 \text{ kJ mol}^{-1}}{12 \text{ nucleons mol}^{-1}} \\ &= \mathbf{7.43 \times 10^8 \text{ kJ/mol nucleons}}\end{aligned}$$

5. Rate of Radioactive Decay and Half-Life

Approach

Radioactive decay processes always follow first order kinetics. The activity (A) of a nuclear decay process is proportional to the number of radioactive atoms present (N), or

$$A/A_0 = k(N/N_0)$$

where k equals the decay or rate constant. The first order integrated rate law equation relates the period over which a sample is observed (t) to the fraction of radioactive atoms present after that amount of time has passed

$$\ln N/N_0 = -kt$$

Another convenient method to find the decay constant is through the **half-life**, $t_{1/2}$. Half-life is defined as the time required for the concentration of a reactant to reach half of its initial value (i.e. $N/N_0 = 0.5$). An equation to determine the half-life from the rate constant can be derived from the first order integrated rate law; the results can be expressed as follow:

$$t_{1/2} = 0.693 / k$$

Example

Some high-level radioactive waste with a half-life $t_{1/2}$ of 200.0 years is stored in underground tanks. What time is required to reduce an activity of 6.50×10^{12} disintegrations per minute (dpm) to a fairly harmless activity of 3.00×10^3 dpm?

Solution

The data provides the initial activity ($A_0 = 6.50 \times 10^{12}$ dpm) and the activity after some elapsed time ($A = 3.00 \times 10^3$ dpm). To find the elapsed time t , first find k from the half-life:

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{200. \text{ years}} = 0.00347 \text{ year}^{-1}$$

With k known, the time t can be calculated:

$$\ln \left[\frac{3.00 \times 10^3}{6.50 \times 10^{12}} \right] = - [0.00347 \text{ year}^{-1}]t$$

$$-35.312 = [0.00347 \text{ year}^{-1}]t$$

$$t = \frac{-35.312}{-[0.00347 \text{ year}^{-1}]}$$

$$= 1.02 \times 10^4 \text{ years}$$

Note that the time unit t and rate constant k must share common units (i.e. years and years^{-1}) for this equation to work properly.