CH 222 Chapter Nine Concept Guide

1. Boyle's Law

Question
A sample of gaseous nitrogen in an automobile airbag has a pressure of 745.0 mm Hg in a 35.00-L bag. If this sample is transferred to a 15.00-L bag at the same temperature, what is the pressure (in mm Hg and atm) of the gas in the new bag?

Approach
Boyle's Law says that pressure is indirectly proportional to volume: $P_1V_1 = P_2V_2$.
We know P_1, V_1, and V_2 and need to solve for P_2.

Solution
$P_2 = P_1V_1/V_2$ = (745.0 mm Hg x 35.00 L)/15.00 L = 1738 mm Hg
1738 mm Hg x (1 atm/760 mm Hg) = 2.29 atm

As expected, the pressure of a gas increases as volume decreases.

2. Charles's Law

Question
The gas volume of CO_2 in a syringe is 25.0 L at 20 °C. What is the final volume of the gas if you hold the syringe in your hand until the temperature reaches 39 °C?

Approach
Charles's Law says that the volume of a gas is directly proportional to the absolute temperature:
$V_1/T_1 = V_2/T_2$. We know T_1, V_1, and T_2. Solve for V_2. Note: Don't forget to convert temperature to Kelvins.

Solution
$V_2 = T_2(V_1/T_1)$ = (312 K)(25.0 mL/293 K) = 26.6 mL

As expected, the volume of a gas increases with an increase in temperature.

3. Ideal Gas Law

Question
If you wanted to use the Ideal Gas Law to solve for volume, how would the equation look?

Approach
Isolate volume from the other variables in the Ideal Gas Law.

Solution
$V = nRT/P$

4. Gas Density and Molar Mass

Question
The density of an unknown gas is 1.50 g/L at STP. What is its molar mass?

Approach
Use the Ideal Gas Law, substituting density for n and molar mass (M; g/mol) for V. Solve for M.

Solution
Density is mass per unit volume and can be used to convert volume into mass: d = m / V. Therefore,

PM = dRT
M = dRT/P
M = [(1.50 g/L)(0.082057 L atm/K mol)(273.15 K)]/1.000 atm = 33.6 g/mol

5. Root-mean-square Speed

A relationship exists among molecular mass, average speed, and temperature. Two gases with different molecular masses must have the same kinetic energy at the same temperature because the average kinetic energy is fixed by temperature. The heavier gas molecules, therefore, must have a lower average speed. Sometimes called the "Maxwell Equation," the **root-mean-square (rms) speed** expresses this idea in quantitative form:

$$\sqrt{\overline{u^2}} = \sqrt{3RT/M}$$

where $\sqrt{\overline{u^2}}$ is called the rms speed, temperature (T) is in Kelvins, M is molar mass, and R is expressed in units related to energy, 8.314 J / K mol.

6. Graham's Law

Gases have the ability to diffuse and effuse. Diffusion is the mixing of two or more substances by random molecular motion; effusion is the movement of gas through a tiny opening in a container into another container.

Thomas Graham (1805 - 1869) studied effusion and found that the rates of effusion of two gases were inversely proportional to the square roots of the molar masses at the same temperature and pressure.

$$\frac{\text{Rate of effusion of gas 1}}{\text{Rate of effusion of gas 2}} = \sqrt{\frac{\text{molar mass of gas 2}}{\text{molar mass of gas 1}}}$$

Therefore, the rate at which a gas will escape through an opening depends on how fast the gas molecules move. This equation is derived from Maxwell's equation.

$$\frac{\text{Rate of effusion of gas 1}}{\text{Rate of effusion of gas 2}} = \frac{\sqrt{\overline{u^2}}\text{ of gas 1}}{\sqrt{\overline{u^2}}\text{ of gas 2}} = \sqrt{\frac{3RT/(\text{M of gas 1})}{3RT/(\text{M of gas 2})}}$$

Canceling out like terms leaves us with the simple equation developed by Graham.

7. Molecular Speeds

Problem

Place the following gases in order of decreasing average molecular speed at 25 °C:

Ar CH_4 N_2 CH_2F_2

Approach

Remember that the average kinetic energy of gas molecules is determined by temperature, thus all these gases have the same average kinetic energy and heavier molecules must move with a slower average speed than lighter molecules at the same temperature.

Solution:

Compound	**Molecular Weights**
CH_4	16.04 g/mol
N_2	28.01 g/mol
Ar	39.90 g/mol
CH_2F_2	52.02 g/mol

8. Molecular Speeds

Problem

Molecular speed is important in the atmospheric diffusion of gases. Rank O_2, O_3, NO_2, and NO in the order of increasing average molecular speed.

Approach

Remember that the average kinetic energy of gas molecules is determined by temperature, thus all these gases have the same average kinetic energy and heavier molecules must move with a slower average speed than lighter molecules at the same temperature.

Solution:

Compound	**Molecular Weights**
O_3	48.00 g/mol
NO_2	46.01 g/mol
O_2	32.00 g/mol
NO	30.01 g/mol

$O_3 < NO_2 < O_2 < NO$

9. Dalton's Law of Partial Pressures

Question
A 5.00 L sample of N_2 at 738 Torr is mixed at constant temperature with 15.5 L of O_2 at 325 Torr. The gaseous mixture is placed in a 10.0 L container. What is the pressure of the mixture?

Approach
We need to apply Boyle's Law to each gas, then use Dalton's Law of partial pressures to find the pressure of the mixture.

Solution
For N_2: P_1 = 738 Torr, P_2 = ?, V_1 = 5.00 L, and V_2 = 10.0 L. When we solve for the unknown, P_2 = 369 Torr.

For O_2: P_1 = 325 Torr, P_2 = ?, V_1 = 15.5 L, and V_2 = 10.0 L. When we solve for the unknown, P_2 = 504 Torr.

According to Dalton's Law of partial pressure, the total pressure of the mixture is:

$$P_{total} = P_{O2} + P_{N2} = 369 \text{ Torr} + 504 \text{ Torr} = 873 \text{ Torr}$$

10. Using Partial Pressures to Measure a Gas Collected over Water

Problem
A volume of 550 mL of gas was collected over water at 21 °C. The atmospheric pressure was 0.980 atm. The vapor pressure of water at this temperature is 0.025 atm. Calculate the partial pressure in atmospheres of the gas collected over water.

Approach
Use the following equation to find the partial pressure of the gas:

$$P_{atm} = P_{H2O} + P_{gas}$$

Solution

$$P_{gas} = P_{atm} - P_{H2O}$$

$$P_{gas} = 0.980 \text{ atm} - 0.025 \text{ atm} = 0.955 \text{ atm}$$

11. Root-mean-square Speed

Problem
Calculate the rms speed of oxygen molecules at 27 °C.

Approach
We need Maxwell's equation to calculate the rms speed. Before plugging in numbers, however, we need to convert M to units of kilograms per mole because R is in units of J / K mol. The necessary conversion factor is: 1 J = 1 kg m^2 / s^2.

Solution

$$\sqrt{\overline{u^2}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3(8.314\ J / K\ mol)(300\ K)}{3.20 \times 10^{-2}\ kg / mol}} = \sqrt{2.34 \times 10^5\ J/kg}$$

To obtain the answer in meters per second, we use the relation I J = 1 kg m^2 / s^2, which means we have

$$\sqrt{\overline{u^2}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{2.34 \times 10^5\ J/kg} = \sqrt{2.34 \times 10^5\ kg\ m^2/kg\ s^2} = 483\ m/s$$

12. Graham's Law

Question
How fast do helium molecules effuse through a barrier relative to oxygen molecules?

Approach
We need Graham's Law to determine the rate differential between helium and oxygen molecules.

Solution

$$\frac{\text{Rate of effusion of He}}{\text{Rate of effusion of } O_2} = \sqrt{\frac{\text{molar mass of } O_2}{\text{molar mass of He}}}$$

$$= \sqrt{\frac{32.00\ g/mol}{4.00\ g/mol}} = 2.83$$

Helium molecules effuse through a barrier 2.83 times faster than oxygen molecules.