Isotopic Abundance Handout and Practice Problems

*Much of the information in this handout was adapted from the handouts "**isotopic abundance - practice problems**" (http://maurermath.weebly.com/uploads/7/0/2/8/7028280/average_atomic_mass_and_percent_abundance.pdf) and "**Calculate the isotopic abundances when given the average atomic weight and the isotopic weights**" (https://www.chemteam.info/Mole/AvgAtomicWt-Reverse.html) - my deep thanks to these authors for their wonderful content!*

The **atomic mass** for each element appearing on the periodic table represents the weighted average of masses for each individual isotope of an element. For example, the atomic mass of carbon is reported as 12.011 amu (atomic mass units). Carbon is composed primarily of two isotopes; carbon-12 and carbon-14. The atomic mass is calculated using *both* the relative abundance and the masses for each of these two isotopes. Using the equation below, the atomic mass for carbon can be calculated.

carbon
6
C
12.011

$$\text{atomic mass} = \left(\text{mass}_1 \times \%_1\right) + \left(\text{mass}_2 \times \%_2\right) + \ldots$$

Carbon-12 accounts for 99.45% of all of the carbon atoms, while carbon-14 only accounts for the remaining 0.55%. Since the carbon-12 isotope is more abundant, its mass is weighted more in the calculation of carbon's atomic mass. The calculation of the atomic mass is shown below.

isotope	% abundance	mass (amu)
carbon-12	99.45	12.000
carbon-14	0.55	14.003

$$\text{atomic mass} = (12.000 \times 0.9945) + (14.003 \times 0.0055)$$

$$\text{atomic mass} = \left(11.934\right) + \left(0.077\right) = 12.011 \text{ amu}$$

Directions: Use the equation for atomic mass to complete the following problems.

1. Argon has three naturally occurring isotopes: argon-36, argon-38, and argon-40. Based on argon's reported atomic mass, which isotope exists as the most abundant in nature? Explain.

2. Copper exists as a mixture of two isotopes. Copper-63 is 69.17% abundant and it has a mass of 62.9296 amu. Copper-65 is 30.83% abundant and it has a mass of 64.9278 amu. Calculate the atomic mass (am) of copper.

3. Calculate the atomic mass (am) of silicon. The three silicon isotopes have atomic masses and relative abundances of 27.9769 amu (92.2297%), 28.9765 amu (4.6832%) and 29.9738 amu (3.0872%).

4. Gallium has two naturally occurring isotopes. The mass of gallium-69 is 68.9256 amu and it is 60.108% abundant. The mass of gallium-71 is 70.9247 amu and it is 39.892% abundant. Calculate the atomic mass (am) of gallium.

5. Bromine has two naturally occurring isotopes. Bromine-79 has a mass of 78.918 amu and is 50.69% abundant. Using the atomic mass reported on the periodic table (79.904), determine the mass of bromine-81, the other isotope of bromine.

6. Calculate the atomic mass of lead. The four lead isotopes have atomic masses and relative abundances of 203.973 amu (1.4000%), 205.974 amu (24.1010%), 206.976 amu (22.1000%) and 207.977 amu (52.3990%).

7. Antimony has two naturally occurring isotopes. The mass of antimony-121 is 120.904 amu and the mass of antimony-123 is 122.904 amu. Using the average mass from the periodic table, calculate the abundance of each isotope.

Answers:

1. Argon has three naturally occurring isotopes: argon-36, argon-38, and argon-40. Based on argon's reported atomic mass, which isotope exists as the most abundant in nature? Explain.

argon has an atomic mass of 39.948 amu, which is closet to **argon-40**, suggesting that it is the most abundant isotope

2. Copper exists as a mixture of two isotopes. Copper-63 is 69.17% abundant and it has a mass of 62.9296 amu. Copper-65 is 30.83% abundant and it has a mass of 64.9278 amu. Calculate the atomic mass (am) of copper.

am = (62.9296 x 0.6917) + (64.9278 x 0.3083) = *63.545645... amu* = **63.55 amu**

3. Calculate the atomic mass (am) of silicon. The three silicon isotopes have atomic masses and relative abundances of 27.9769 amu (92.2297%), 28.9765 amu (4.6832%) and 29.9738 amu (3.0872%).

am = (27.9769 x 0.922297) + (28.9765 x 0.046832) + (29.9738 x 0.030872) = *28.0853895... amu* = **28.085 amu**

4. Gallium has two naturally occurring isotopes. The mass of gallium-69 is 68.9256 amu and it is 60.108% abundant. The mass of gallium-71 is 70.9247 amu and it is 39.892% abundant. Calculate the atomic mass (am) of gallium.

am = (68.9256 x 0.60108) + (70.9247 x 0.39892) = *69.723080... amu* = **69.723 amu**

5. Bromine has two naturally occurring isotopes. Bromine-79 has a mass of 78.918 amu and is 50.69% abundant. Using the atomic mass reported on the periodic table (79.904), determine the mass of bromine-81, the other isotope of bromine.

79.904 = (78.918 x 0.5069) + ($mass_2$ x 0.4931) → 39.9005 = $mass_2$0.4931 → $mass_2$ = *80.91766... amu* = **80.92 amu**

6. Calculate the atomic mass of lead. The four lead isotopes have atomic masses and relative abundances of 203.973 amu (1.4000%), 205.974 amu (24.1010%), 206.976 amu (22.1000%) and 207.977 amu (52.3990%).

atomic mass = (203.973 x 0.014000) + (205.974 x 0.241010) + (206.976 x 0.221000) + (207.977 x 0.523990) = *207.21169... amu* = **207.22 amu**

7. Antimony has two naturally occurring isotopes. The mass of antimony-121 is 120.904 amu and the mass of antimony-123 is 122.904 amu. Using the average mass from the periodic table, calculate the abundance of each isotope.

121.760 = 120.904(x) + 122.904(1-x) → 121.760 = 120.904x + 122.904 - 122.904x → -1.144 = -2.000x → x = 0.5720

^{121}Sb = 57.20 %, and **^{123}Sb** = 100 - 57.20 = **42.80 %** *(more on these kinds of problems in the next section!)*

Calculate the Isotopic Abundances When Given the Average Atomic Weight and the Isotopic Weights

Let's start by calculating the **average atomic weight** for nitrogen. Nitrogen-14 has a mass of 14.003074 amu and an abundance of 99.63%, while nitrogen-15 has a mass of 15.000108 amu and an abundance of 0.3700%, so:

$$(14.003074)\ (0.9963) + (15.000108)\ (0.003700) = \mathit{14.006763...} = \mathbf{14.007\ amu}$$

The solution is laid out like this:

> (exact weight of isotope #1) (abundance of isotope #1) + (exact weight of isotope #2) (abundance of isotope #2) = **average atomic weight** of the element

Note how the abundance values are given in decimal form, so make sure to divide the percentage values by 100%.

Solving equations like this is possible because the sum of the percent abundances of the two isotopes add up to 100% (or, since we use decimal abundances in the calculation, 1.00). So, the trick is to express both abundances using only one unknown. This is how to do so using the above nitrogen example.

<u>Problem #1</u>: Nitrogen is made up of two isotopes, N-14 and N-15. Given nitrogen's atomic weight of 14.007, what is the percent abundance of each isotope?

Here's the *solution*:

$$(14.003074)\ (x) + (15.000108)\ (1 - x) = 14.007$$

Notice that the abundance of N-14 is assigned 'x' and the N-15 is 'one minus x.' The two abundances always add up to one (or, if you prefer, 100%)

Note: sometimes the exact weights of the isotopes are not provided in the problem. However, in this Internet era, it is easy to look up the values online. If you do not have access to the internet, notice that the mass numbers (14 and 15) are quite close to the exact values. The consequence of using 14 and 15 rather than the exact values is that we will get slightly approximate answers for the two abundances (and worse sig figs.)

For example, I might have this:

$$(14)\ (x) + (15)\ (1 - x) = 14.007$$

Solving gives:

$$14x + 15 - 15x = 14.007$$

After some more simple algebra,

we get: $x = 15 - 14.007 =$

0.993

so

$$1 - x = 0.007$$

You may want to compare them to the more exact abundances which are in the equation set up at the top of this page.

<u>Problem #2a</u>: Copper is made up of two isotopes, Cu-63 (62.9296 amu) and Cu-65 (64.9278 amu). Given copper's atomic weight of 63.546, what is the percent abundance of each isotope?

Solution:

1) Write the following equation:

$$(62.9296)(x) + (64.9278)(1 - x) = 63.546$$

Once again, notice that 'x' and 'one minus x' add up to one.

2) Solve for x:

$62.9296x + 64.9278 - 64.9278x = 63.546$
$-1.9982x = -1.3818$ *divide both sides by -1.9982*
$x = -1.3818 / -1.9982 =$ *0.691522...* = **0.6915** (the decimal abundance for Cu-63, or **69.15 %**)
Cu-65 = 1 - 0.6915 = **0.3085**, or **30.85%**
check: both isotopes should equal 100% (or be very close to 100%!), so:
69.15 + 30.85 = **100.00%** *good!*

Note that this calculation technique works only with two isotopes. If you have three or more, there are too many variables and not enough equations. Also note the four sig figs: this stems from 63.546 - 64.9278, the answer should stop at the thousandths position, even though we use all the values in the calculation until the end of the problem.

<u>Problem #2b</u>: Chlorine is made up of two isotopes, Cl-35 (34.969 amu) and Cl-37 (36.966 amu). Given chlorine's atomic weight of 35.453 amu, what is the percent abundance of each isotope?

Solution:

1) Write the following equation:

$$(34.969)(x) + (36.966)(1 - x) = 35.453$$

2) Solve for x:

$34.969x + 36.966 - 36.966x = 35.453$
$-1.997x = -1.513$
$x = -1.515 / -1.997 =$ *0.757636...* = **0.7576** (the decimal abundance for Cl-35, or **75.76%**)
Cl-37 = 1 - 0.7576 = **0.2424**, or **24.24%**
check: both isotopes should equal 100% (or be very close to 100%!), so:
75.76 + 24.24 = **100.00%** *good!*

<u>Problem #3</u>: A sample of naturally occurring silicon consists Si-28 (amu = 27.9769), Si-29 (amu = 28.9765) and Si-30 (amu = 29.9738). If the atomic mass of silicon is 28.0855 and the natural abundance of Si-28 is 92.23%, what are the natural abundances of Si-29 and Si-30?

Solution:

1) Set up a system of two equations in two unknowns:

Let x = isotopic abundance of Si-28 (as a decimal)
Let y = isotopic abundance of Si-30 (as a decimal)

Therefore:

(27.9769) (0.9223) + (28.9765) (x) + (29.9738) (y) = 28.0855, *and*

0.9223 + x + y = 1.000 *(all abundances must equal 100%)*

1) Rearrange the second equation to:

y = 1.000 − 0.9223 − x = **0.0777 − x**

2) Substitute into the first equation and solve:

(27.9769) (0.9223) + (28.9765) (x) + (29.9738) (0.0777 − x) = 28.0855

This is one equation with one unknown (x), so time for some algebra!
Note the 0.0777 will make the final x value 3 sig figs

25.80309 + 28.9765x + 2.32896 − 29.9738x = 28.0855
-0.9973x = -0.04655
x = *0.046676...* = **0.0467** (the decimal abundance for **Si-29**, or **4.67%**)
Si-30 = 0.0777 - x = 0.0777 - 0.0467 = **0.0310**, or **3.10%**
check: the three isotopes should equal 100% (or be very close to 100%!), so:
92.23 + 4.67 + 3.10 = **100.00%** *good!*

Problem #4: Determine the percent abundance for Fe-57 and Fe-58, given the following data and the atomic mass of iron (55.845):

Isotope	Atomic Weight	Percent Abundance
Fe-54	53.9396	5.845
Fe-56	55.9349	91.754
Fe-57	56.9354	???
Fe-58	57.9333	???

Solution:

1) Assign the percent abundance of Fe-57 to the variable 'x' and Fe-58 to the variable 'y'

2) We need to get the percent abundance for Fe-58 in terms of x. Like this:

$1 = 0.05845 + 0.91754 + x + y$; *solve for y*
$y = 1 - (0.05845 + 0.91754 + x) = 1 - (0.97599 + x)$ *and*
$y = 1 - 0.97599 - x = \mathbf{0.02401 - x}$

We will use $y = 0.02401 - x$ in the full equation below.

3) Set up (and solve) the following equation:

$(53.9396)(0.05845) + (55.9349)(0.91754) + (56.9354)(x) + [(57.9333)(0.02401 - x)] = 55.845$
$3.15277 + 51.32251 + 56.9354x + [1.39098 - 57.9333x] = 55.845$
$55.86626 - 0.9979x = 55.845$
$-0.9979x = -0.02126$
$x = -0.02126 / -0.9979 = 0.0213047... = \mathbf{0.02130}$ (the decimal abundance for **Fe-57**, or or **2.130%)**
Fe-58 $= y = 0.02401 - x = 0.02401 - 0.02130 = \mathbf{0.00271}$, or **0.271%**
check: the four isotopes should equal 100% (or be very close to 100%!), so:
$5.845 + 91.754 + 2.130 + 0.271 = \mathbf{100.00\%}$ *good!*