CH 221 Introductory Mathematics Concept Guide

Right from the beginning of your study of chemistry you will face the task of solving problems. The goal of this section is to provide you with a review of the problem-solving skills that you will need.

1. Algebra Basics

a. Variables

Algebra is used to solve problems where some of the variables are known, but others are not. The unknowns are represented by letters. Problems are represented as equations and solved by finding the value of the unknown that makes the equation a true statement.

When solving equations, we can perform any operation (addition, subtraction, multiplication, or division) as long as we appropriately do the same thing to both sides of the equation.

When a number and a variable are written together, multiplication is indicated. 4x means four times x. Like terms can be added or subtracted.

$3x + 4x = 7x.$

Unlike terms cannot be combined.

$3x + 4x + 2y = 7x + 2y$

There may be situations for which you will be required to add algebraic equations together. To do this, line up the equal signs and add each side of the equations separately.

b. Notation Conventions

There are certain notation conventions that you may come across in example or solved problems in your chemistry work. Understanding these rules will not only help you follow problem solutions, but will help you keep track of your math as you solve problems on your own.

To avoid confusion with the variable x, a multiplication dot (·) is frequently used instead of the times symbol (x). When equations contain parentheses, do the operation within the parentheses first. When parentheses are written next to each other, or a number is written directly outside parentheses, multiplication is indicated.

Subtracting a positive number is equivalent to adding a negative number. Subtracting a negative number is equivalent to adding a positive number.

When multiplying numbers, multiplying numbers of opposite sign gives a negative product, and multiplying numbers of the same sign results in a positive product.

2. Dimensional Analysis

Dimensional analysis is a systematic way of solving numerical problems by the conversion of units. Frequently, your data will be recorded in one unit, but it will be necessary to do the calculation using a different unit of measurement. This means you should multiply the number you wish to convert by a conversion factor to produce a result in the desired unit. This way the units in the denominator cancel the units of the original data, leaving the desired units.

For example, if you pour a quarter liter of water from a full 500 mL container, how much remains in the container? To solve this problem, we need to determine how much water was poured out in milliliters (mL). We can use the conversion factor (1000 mL/1L).

0.25 L (1000 mL/1L) = 250 mL
500 mL - 250 mL = 250 mL left in the container

Question
A 200. cm^3 volume of liquid weighs 226 g. What volume of this liquid will weigh 5.0 g?

Solution
(200. cm^3/226 g)(5.0 g) = 4.4 cm^3

Question
What is the mass in kilograms of a 7.00 cm^3 piece of lead? The density of lead is 11.3 g/cm^3.

Solution
(7.00 cm^3 Pb)(11.3 g/cm^3 Pb)(1 kg/1000 g) = 0.0791 kg Pb

3. Quadratic Equation

The most likely place you will encounter quadratic equations is in the chapter on chemical equilibria, during the second semester of General Chemistry. Solving quadratic equations may seem overwhelming at first, but if you take one step at a time you will quickly see that it requires no more algebra then you have already learned.

A quadratic equation with one variable, x, is in the form:

$ax^2 + bx + c = 0 \qquad a \neq 0$

The coefficients a, b, and c may be either positive or negative numbers. If no coefficient is written, assume it to be one. The two roots of the equation can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When you have solved a quadratic expression, you should always check your values by substitution into the original equation.

How do you know which of the two answers is the correct one? You have to decide in each case which root has physical significance. In chemistry, it is often the case that the negative value is not significant.

4. Logarithms

The logarithm of a positive number N to a given base b (written $\log_b N$) is the exponent of the power to which b must be raised to produce N. The most common log is base 10, usually written log N. In calculus, the most useful system of logarithms is the natural system in which the base is an irrational number symbolized by the letter e.

Despite the different bases of the two logarithms, they are used in the same manner. Our discussion will focus on common logarithms. A common logarithm is the power to which you must raise ten to obtain the number, $\log 10^x = x$. For example, the log of 100 is 2, since you must raise 10 to the second power to obtain 100.

To obtain the common log of a number other than a simple power of 10, you will need a calculator. For example, log 5.15 = 0.7118, which means that $10^{0.7118} = 5.15$.

Operations involving logarithms follow the same rules as those for exponents.

General laws of logarithms

- The log of the product of two or more positive numbers is equal to the sum of the logs of several numbers. For example, log (a x b)= log a + log b.
- The log of the quotient of two positive numbers is equal to the log of the dividend minus the log of the divisor. For example, log a/b = log a - log b.
- The log of a power of a positive number is equal to the log of the number, multiplied by the exponent of the power. For example, log a^n = n log a.
- The log of a root of a positive number is equal to the log of the number, divided by the index of the root. For example, log $\sqrt[n]{a}$ = (log a)/n
- The log of 1 = 0, log of numbers >1 are positive
- You can't have logs of negative numbers or of zero.

5. Significant Figures

The precision of a measurement indicates how well several determinations of the same quantity agree. In the laboratory, chemists attempt to set up experiments so that the greatest possible accuracy can be achieved. For each individual experiment, several measurements are usually made and their precision determined. Usually, better precision is taken as an indication of better experimental work. A calculated result can be no more precise than the least precise piece of information that went into the calculation. This is why the rules of significant figures are used.

Rule 1

To determine the number of significant figures in a number, read the number from left to right and count all the digits, starting with the first digit that is not zero. If the last digit of a number does not contain a decimal point, then the number of significant figures is equal to the number of non-zero digits in the number. Zeros to the left of 1 only locate the decimal point. This is clearer when it is written in scientific notation.

Rule 2

When adding or subtracting, the number of decimal places in the answer should be equal to the number of decimal places in the number with the fewest places.

Rule 3

When multiplying or dividing, the number of significant figures in the answer should be the same as the number with the fewest significant figures.

Rule 4

When a number is rounded off (the number of significant figures is reduced), the last digit is increased by 1 only if the following digit is 5 or greater. When calculating, you should do the calculation using all of the digits allowed by the calculator and round off only at the end of the problem. Rounding off in the middle of the problem can cause errors.

Question

How many significant figures are there in the numbers

(a) 57 (b) 62.9 (c) 20.000 (d) 0.003 (e) 0.0403 (f) 0.04030?

Solution

(a) Beginning with the 5 and counting gives two significant figures.
(b) Beginning count with the 6 gives three significant figures.
(c) All zeros here are significant. There are five significant figures.
(d) None of the zeros here are significant, giving one significant figure.
(e) The zeros to the left are insignificant, but the zero in the middle is a significant digit, giving three significant figures.
(f) The zero at the end and the zero in the middle are significant digits, giving four significant figures.

Problem

Perform the following calculations and express the answers to the proper number of significant figures:

(a) 14 - 0.052 (b) 32.1/3.21 (c) 13.79/0.0002 (d) (0.0801)(10)

Solution

(a) 14 (b) 10.0 (c) 70,000 (d) 0.801

(Note: the exact factor of 10 does not limit the number of significant figures in the answer.)

6. Percents and Fractions

Fractions

A fraction is a part of a whole. In its simplest form, the value of a fraction is less than one. An example commonly used to explain fractions is a pie: cut the pie into eight slices, then eat two. Two-eighths of the pie is gone. There are six pieces left, so six-eighths (6/8) of the pie is left.

a. Reducing Fractions

Imagine you have three pies cut into sections: one into fourths, one into eighths, and one into sixteenths. Take one piece from the first pie, two from the second, and four from the third. How much has been eaten from each pie? From pie one: one-fourth (1/4), from pie two: two eighths (2/8), and from pie three: four-sixteenths (4/16). The same amount, however, has been taken from each pie: one quarter (1/4). As you can see, it is frequently more convenient to talk about fractions in the most reduced, or simplest terms.

To reduce a fraction:

1. Find a number that divides evenly into the numerator and the denominator.
2. Check to see if another number goes in evenly. Repeat until the fraction is reduced as far as possible.

Example: 48/64

(48/8)/(64/8) = 6/8

(6/2)/(8/2) = 3/4

b. Converting Between Improper Fractions and Whole/ Mixed Numbers

An improper fraction is one in which the numerator is larger than the denominator. For example, if you were told you had six-fourths (6/4) of a pie left, you would know that you had one whole pie (4/4) plus one-half of a pie (2/4). It can be more useful to express the fraction as a mixed number, a number containing a whole number and a fraction. Thus rather than 6/4 of a pie, you have 1 1/2 pies.

To change an improper fraction into a mixed number:

1. Divide the denominator into the numerator.
2. Write the remainder as a fraction over the original denominator.
3. Reduce the remaining fraction.

c. Changing a Mixed Number into an Improper Fraction

When carrying out mathematical operations, it is usually necessary to work with improper fractions rather than mixed numbers. To change a mixed number into an improper fraction:

1. Multiply the whole number by the denominator.
2. Add the numerator to the product.
3. Place the sum in the numerator, over the original denominator.

d. Multiplying and Dividing Fractions

To multiply fractions, simply multiply the numerators together and multiply the denominators together. When multiplying fractions, you can cross reduce: reduce as you normally would a fraction, but use the numerator of one fraction and the denominator of the other. This will save you from having to reduce the fraction product. When multiplying a fraction by a whole number, write the whole number as a fraction over one, and multiply as usual.

Example: (15/7) x (3/5)

1. divide both 15 and 5 by 5
2. (3/7) x (3/1) = 9/7

To divide by a fraction, invert the fraction to the right of the division sign and multiply. Example: 1/2 / 1/8

1. (1/2)(8/1) = 8/2 = 4

e. Adding and Subtracting Fractions

Many problems will require mathematic manipulations of fractions. To continue with the pie example, if you have 3/8 of one pie and 1/4 of another pie, how much pie do you have?

To add or subtract fractions, they must have the same denominator. Then you simply add the numerators. Thus, we have 3/8 of one pie and 2/8 of another, leaving us with 5/8 of a pie.

If the fractions you want to add do not have a common denominator, you must change one or both of the fractions. You can do so by multiplying one or more of the fractions by an expression equivalent to one.

f. Decimals

In one of our earlier examples, we determined that a quarter of the pie had been eaten. We expressed one quarter as a fraction, 1/4. It can also be expressed as a decimal, 0.25, which is read as twenty-five hundredths. Any fraction can be expressed as a decimal by dividing the numerator by the denominator.

g. Rounding Off

When a fraction's denominator does not divide evenly into the numerator, the decimal equivalent can be long and too cumbersome to work with. For example, 1/3 = 0.3333333... The decimal is infinite, so we will want to use an abbreviated version for our records and calculations. An acceptable equivalent, rounded to two decimal places, is 0.33. When rounding off, increase the last digit retained by one if the following digit is greater than or equal to 5. Leave the last digit unchanged if the following digit is less than 5.

Percents

The percent symbol (%) means per hundred. 15% is equivalent to 15/100 or 0.15. Thus, when a quarter of our pie was eaten, 0.25, or 25% was gone. Any percentage may be expressed in decimal form by dividing by 100 and dropping the percent symbol. For example, 52.3% = 52.3/100 = 0.523. To calculate percentage, you must change the percent to decimal form (divide the percent by 100) and multiply.