

Problem Solving - The Dimensional Analysis Method

Dimensional analysis, also known as the **factor-label method**, is a problem solving technique using units (labels) and conversion factors. **Units** tell us the type of measurement being made; for example, "5.2 cm" has a unit (cm) which tells us the measurement to be made is length. **Conversion factors** (also known as "equalities") are fractions that relate two kinds of units; for example, "6.0 cm / s" tells us that for every 1 second that passes (time), 6.0 cm are covered (length). Notice that "6.0 cm s⁻¹" is the same as "6.0 cm / s" in unit notation.

Most problems ask a question whose answer is a number with its unit. Problems also give information that contains numbers with their units. Multiply the information by conversion factors so that all units cancel *except* the units needed in the answer. (A unit in the numerator may be cancelled by placing the same unit in the denominator of the neighboring conversion factor. Conversely, a unit in the denominator may be cancelled by placing the same unit in the numerator of the neighboring conversion factor.)

- Numbers and units are considered separately.
- Multiply by as many conversion factors as necessary.
- Common conversion factors may or may not be supplied with the problem.

Examples:

1. How many hours are in 6 days?

$$6 \text{ days} * \frac{24 \text{ hours}}{1 \text{ day}} = 144 \text{ hours}$$

2. How many seconds are in 5 hours?

$$5 \text{ hours} * \frac{60 \text{ minutes}}{1 \text{ hour}} * \frac{60 \text{ seconds}}{1 \text{ minute}} = 18,000 \text{ seconds}$$

3. How many feet per second is 5 miles per hour?

$$\frac{5 \text{ miles}}{1 \text{ hour}} * \frac{1 \text{ hours}}{60 \text{ minutes}} * \frac{1 \text{ minute}}{60 \text{ seconds}} * \frac{5280 \text{ feet}}{1 \text{ mile}} = 7.3 \frac{\text{feet}}{\text{second}}$$

A common **variation** occurs when more than one piece of information is provided with the problem. In these cases, start with the information that contains the same type of units as the answer. (For example, if length units are needed in the numerator of the answer, use the information that contains length units and arrange it so that those length units are in the numerator.) Next, multiply by conversion factors so that unwanted units cancel.

Test Yourself:

1. How many gallons of milk does a family drink in 5 days if they drink 4 pints per day? *Answer:* 2.5 gallons
2. How many minutes will it take an automobile traveling 60 miles per hour to travel a distance of 400 miles? *Answer:* 400 minutes

Using Scientific Notation

Scientific Notation is used by scientists to express very large and very small numbers in a compact fashion.

To express a number in scientific notation, we rewrite the quantity as a number (between 1 and 10) times 10 raised to a power (exponent) that tells us how we moved the decimal point.

- Multiply the number by 10^0 ($10^0 = 1$)
- Move the decimal point to give a number between 1 and 10
- Every time we shift the decimal point to the left by one place we increase the value of the exponent by one
- Every time we shift the decimal point to the right by one place we reduce the value of the exponent by one

Example: Write 120,000 in scientific notation.

$$120,000 = 120,000 * 10^0 = 1.2 * 10^5$$

Example: Write 0.0000012 in scientific notation.

$$0.0000012 = 0.0000012 * 10^0 = 1.2 * 10^{-6}$$

To express a number that is written in scientific notation as a non-exponential quantity:

- Move the decimal point the same number of places as the value of the exponent and eliminate the exponential part of the number.
- If the exponent is positive, we move the decimal to the right to the same number of places as the value of the exponent. The result should be a number *greater than 1* **unless** the original number is negative.
- If the exponent is negative, we move the decimal to the left to the same number of places as the value of the exponent. The result should be a number *less than 1* **unless** the original number is negative.

Example: Write $1.23 * 10^6$ in non-exponential form.

$$1.23 * 10^6 = 1,230,000$$

Example: Write $1.11 * 10^{-5}$ in non-exponential form.

$$1.11 * 10^{-5} = 0.0000111$$

PROBLEM SOLVING - The Dimensional Analysis Method

Name:

Directions: Use this form to complete this assignment. Show all work in a neat and orderly fashion using the dimensional analysis method. Underline or box your final answer with the correct significant figures and units. Use scientific notation if appropriate.

Helpful Conversion Factors:

Mass: 453.6 g = 1 lb, and 28.34 g = 1 ounce
Volume (wet): 0.9464 L = 1 qt = 2 pints
1 gallon = 3.7856 L = 4 qt = 128 fluid ounces
1 mL = 1 cm³
Distance: 2.54 cm = 1 in (exact)
1.609 km = 1 mile
12 inches = 1 foot (exact)

1. A piece of copper weighs 17.62 lb. What is the weight in kg?
2. The average female has 4500 mL of blood. What is this volume in gallons?
3. The distance from the earth to the moon is 3.9×10^5 km. What is the distance in feet?
4. The fastest pitch in major league baseball was thrown at 100.8 miles per hour. What was the speed in meters per sec?
5. The human heart pumps blood at the rate of 6.8 fl oz/sec. How many gal/hr does the heart pump? (Note: Ounces and fluid ounces are not the same units)

6. What is the water capacity in cubic meters of a rectangular swimming pool, dimensions: 12.0 ft long; 6.00 ft wide; 8.00 ft deep?

7. Osmium is the densest metal. What is its density (g/cm^3) if 50.00 g occupies 2.22 cc? (1 cc = 1 mL)

8. If your blood has an average density of 1.05 g/mL at 20 $^\circ\text{C}$, how many pounds of blood do you lose when you donate 1.0 pint of blood?

9. The density of ethanol is 0.789 g/mL . How many quarts would 5.00 lbs occupy?

10. How many pints is exactly one pound of table salt (density = 2.16 g/cm^3)?

11. Milk is pasteurized at 145 $^\circ\text{F}$. What is this in $^\circ\text{C}$? In Kelvin?

12. An oven to cook pizza is set at 275 $^\circ\text{C}$. What is this in $^\circ\text{F}$? Kelvin?

13. If a bumblebee weighs 0.0022 kg, how many bees weigh 5.0 lb?

14. A copper alloy is found to be 38.7% copper by mass. How many kilograms of copper are present in a 30.2 pound sample?

15. A cylindrical rod of copper is 3.21 inches long and has a diameter of 3.2 in. What is the mass in ounces? The density of copper is 8.93 g/cm^3 . [Hint: Find volume of cylinder first! $V = \pi r^2 L$]

16. Metric conversions: **All answers must be in correct scientific notation.** Helpful metric prefixes and conversions: micro (μ , 10^{-6}), nano (n, 10^{-9}), pico (p, 10^{-12}), femto (f, 10^{-15}), Mega (M, 10^6)

6.50 m = _____ cm 7.25 cm = _____ km 413 ng = _____ mg

33 kg = _____ mg 4.39 pg = _____ mg 23.1 cm = _____ Mm

43 mg = _____ g 62 μL = _____ dL 6.3 fm = _____ cm

22.5 mm = _____ nm 7.6 cm = _____ nm 352 kg = _____ dg

| Prefix | Base | Symbol | Acronym | Example with mass |
|--------------|-----------|--------|----------------|-------------------|
| kilo | 10^3 | k | <i>kiss</i> | kilogram, kg |
| hecto | 10^2 | h | <i>him/her</i> | hectogram, hg |
| deka | 10^1 | da | <i>dearly</i> | dekagram, dag |
| <i>base</i> | 10^0 | base | <i>but</i> | gram, g |
| deci | 10^{-1} | d | <i>don't</i> | decigram, dg |
| centi | 10^{-2} | c | <i>call</i> | centigram, cg |
| milli | 10^{-3} | m | <i>me</i> | milligram, mg |

Also: micro (μ) = 10^{-6} and nano (n) = 10^{-9}

Example: There are 10^3 grams in one kilogram,
or there are 10^{-3} kilograms in one gram

Example: There are 10^{-2} grams in one centigram,
or there are 10^2 centigrams in one gram

Example: There are 10^2 liters in one hectoliter,
or there are $(10^{-2} \text{ hL/L})(10^{-2} \text{ L/cL}) = 10^{-4} \text{ hL / cL}$

